



UNIVERSITÄT ZU LÜBECK
INSTITUT FÜR MEDIZINISCHE
BIOMETRIE UND STATISTIK



Introduction to Bayesian Statistics for Physicians

Reinhard Vonthein, Joachim Gerß

Reinhard.Vonthein@uni-luebeck.de

- ▶ All physicians licenced to practice medicine in Germany receive weekly *Deutsches Ärzteblatt*
- ▶ IF 7.7, circulation 390'000, 5 Mio visits/month
- ▶ Peer reviewed
- ▶ Series on the Evaluation of Scientific Publications
- ▶ Statistician in Editorial Board
- ▶ *Review Article*

MEDICINE

Introduction to Bayesian Statistics

Part 36 of a Series on the Evaluation of Scientific Publications

Joachim Werner Otto Gerß*, Reinhard Vonthein*

- ▶ \leq 2300 words main text
- ▶ General interest
- ▶ Readership: practitioners
- ▶ \leq 6 authors
- ▶ \leq 5 figures and tables

- ▶ Quantity, everybody is familiar with: **body height**
- ▶ Effect, one can relate to: Men taller than women by >1 SD
- ▶ Population distribution known here
- ▶ Populations known to differ elsewhere
- ▶ Natural mixture by sex

- ▶ Approximation for one sex by normal-normal model, SD known?
- ▶ Robust mixture prior extremely defensible
- ▶ >2300 words

- ▶ Quantity, everybody is familiar with: **treatment success**
- ▶ „Effect“: Percentage
- ▶ Parallel to diagnostic test
- ▶ Easily told story of registry and single arm

- ▶ Beta-binomial model
- ▶ Parameters = observed natural numbers
- ▶ Prior-data mismatch plausible

- ▶ Lots of densities as vector graphic or pdf
- ▶ Scope of applications without examples

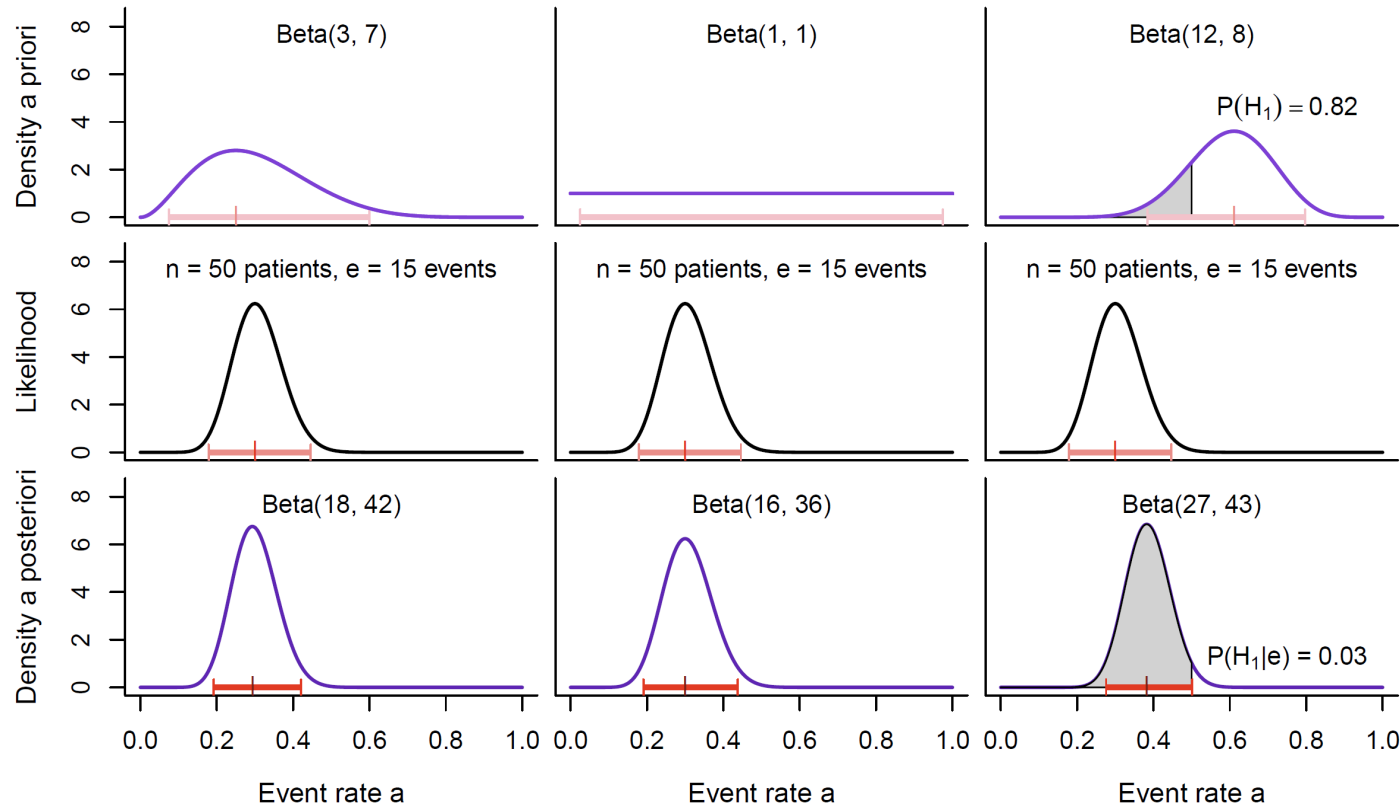
Second try (24.08.2024)



(i) Informative

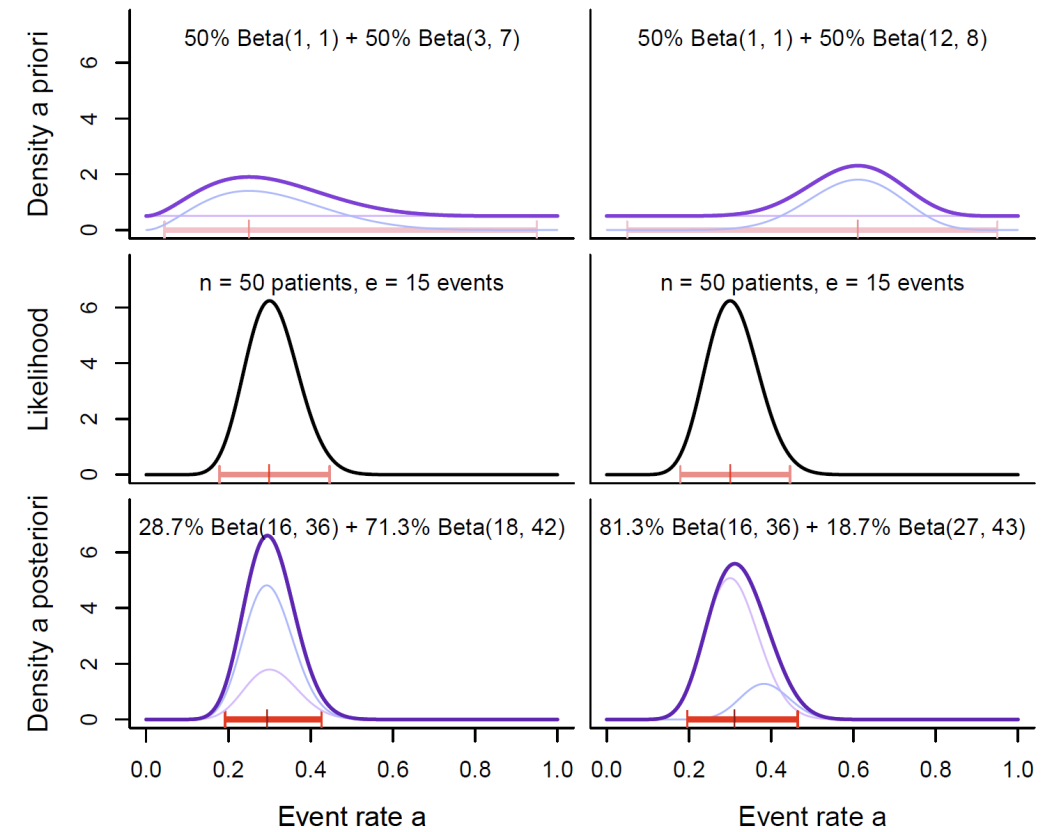
(ii) Non-informative

(iii) Prior-data mismatch



Robust informative

Robust with outdated prior information



Editor's requirements (29.11.2024)

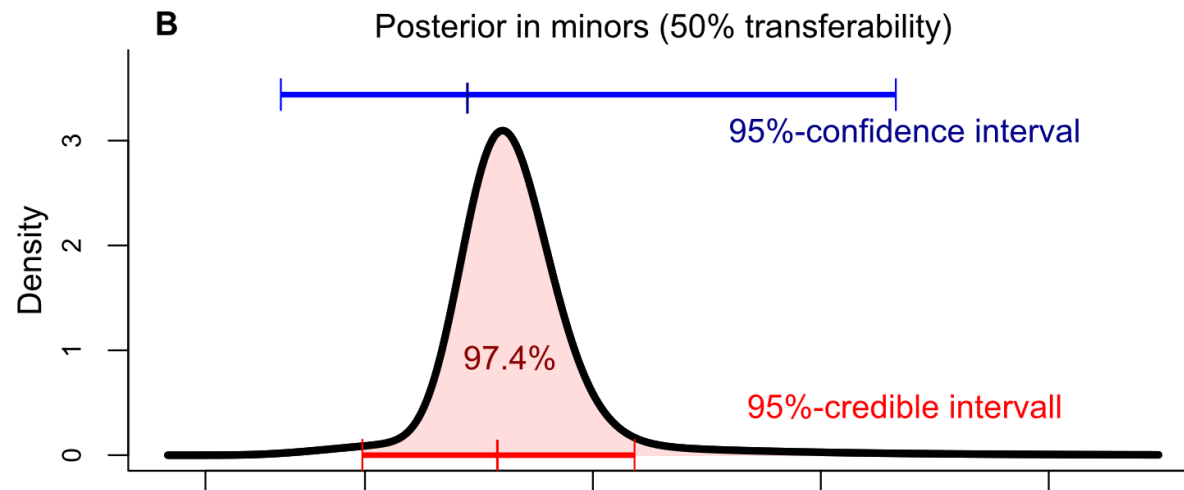
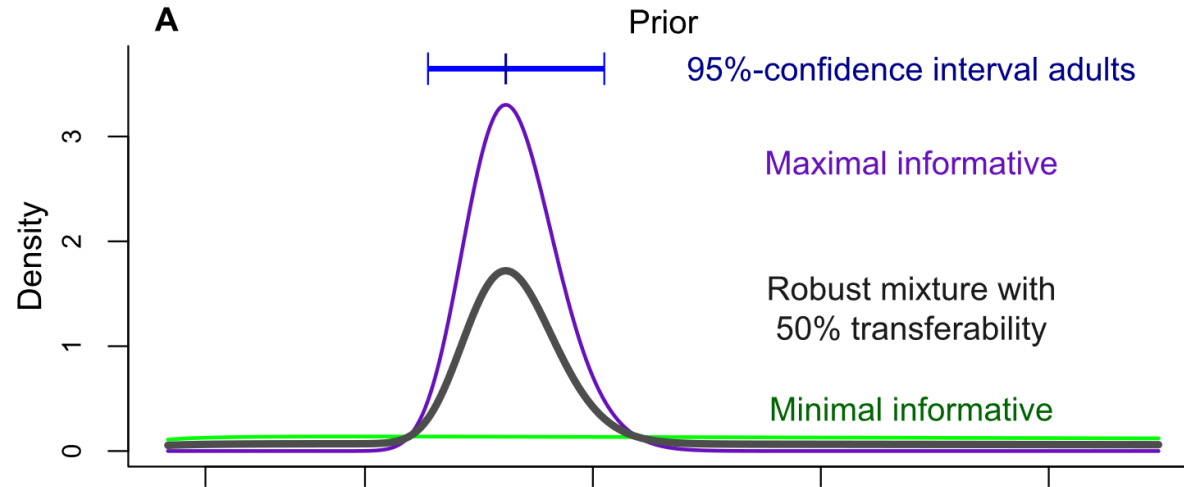


- ▶ No prerequisites
- ▶ Explain jargon
- ▶ Essentials for appraisal of articles
- ▶ Comparison to frequentist statistics
- ▶ Supplements
- ▶ Formats: PowerPoint, Word, Excel

- ▶ Helpful peer review
- ▶ $\text{Bin}(3, 7)$ is a formula

- ▶ Simplification: Prior for effect measure **OR**
- ▶ Real example of drug market authorization
- ▶ Extrapolation from adults to minors \neq meta-analysis
- ▶ Boxes
 - ▶ Glossary
 - ▶ Examples in different fields of medicine
 - ▶ Q&A
- ▶ = 2300 words

Third try (20.01.2025)



“ ...

A robust mixture prior (RMP) [Schmidli, 2014] was used for the treatment effect in paediatrics δ_p . A mixture of Normals with two components was used: an informative one based on the efficacy results from adults and a vague one to enable the prior to downweight the information from adults in case of conflict between the observed results in the two populations. That is, the prior for δ_p has the form

$$p(\delta_p) = w \cdot p_A(\delta_p) + (1 - w) \cdot p_v(\delta_p) \quad (1)$$

Where $p_A(\delta_p)$ is the component containing the information from adults and $p_v(\delta_p)$ is the vague component with variance scaled to represent information equivalent to one subject per arm.

... “

“
...

$$y_p \sim N(\delta_p, s_p^2)$$
$$\delta_p \sim w \cdot N(y_A, s_A^2) + (1 - w) \cdot N(0, m \cdot s_p^2)$$

Where y_p is the estimated treatment effect in the paediatric study and s_p is the corresponding SE, which is assumed to be known.

The prior distribution for δ_p is a mixture of the form in expression (1), with $p_A(\delta_p) = N(y_A, s_A^2)$, where y_A, s_A are the estimated treatment effect and SE, respectively, from the combined adult studies, and the vague component is $p_v(\delta_p) = N(0, m \cdot s_p^2)$; w is the prior weight given to the information from adults.

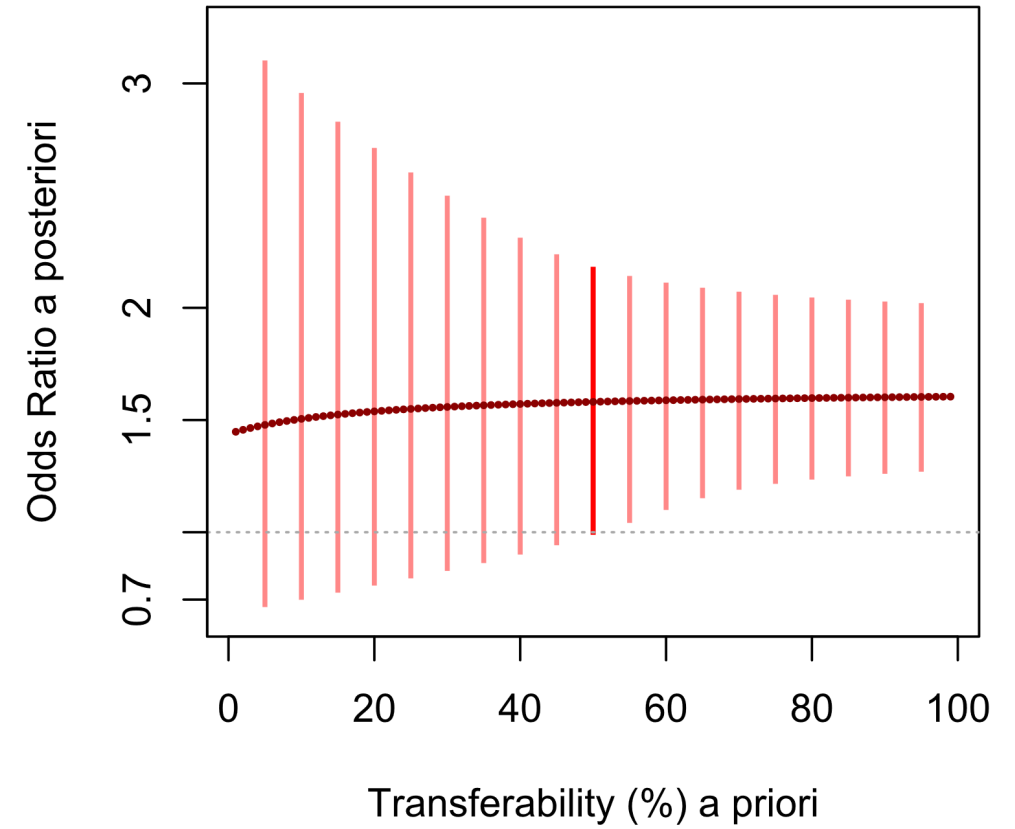
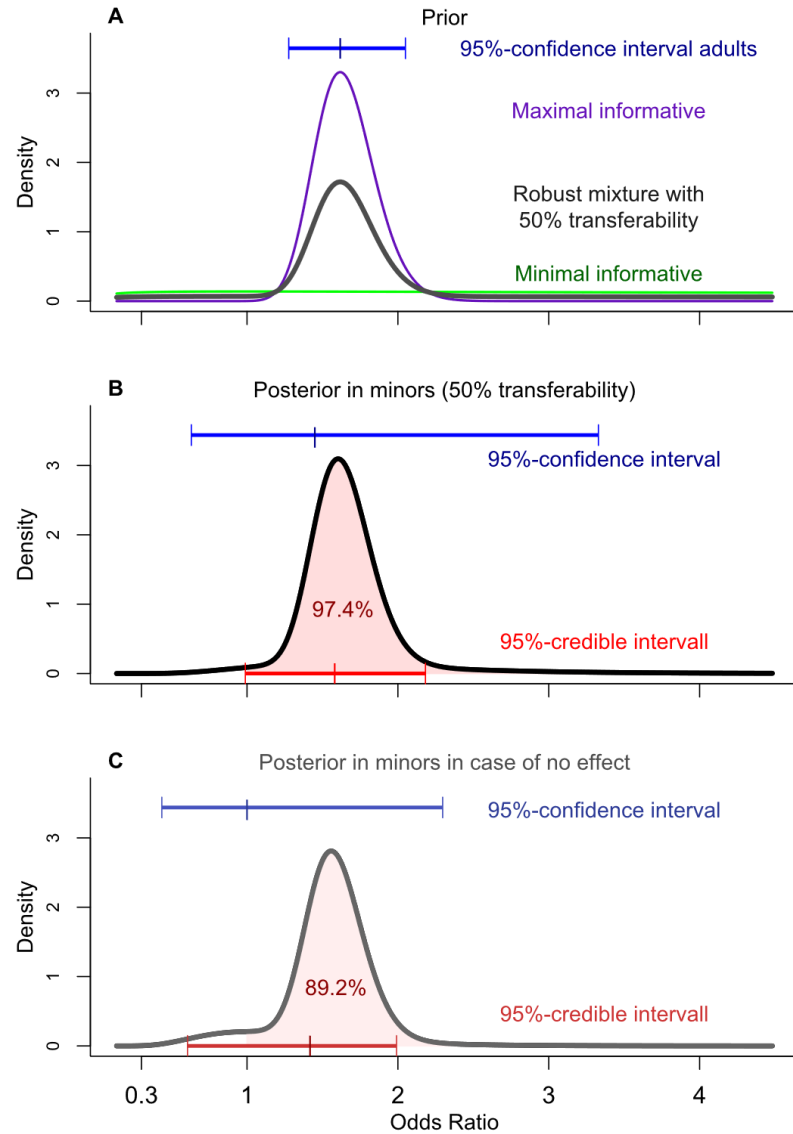
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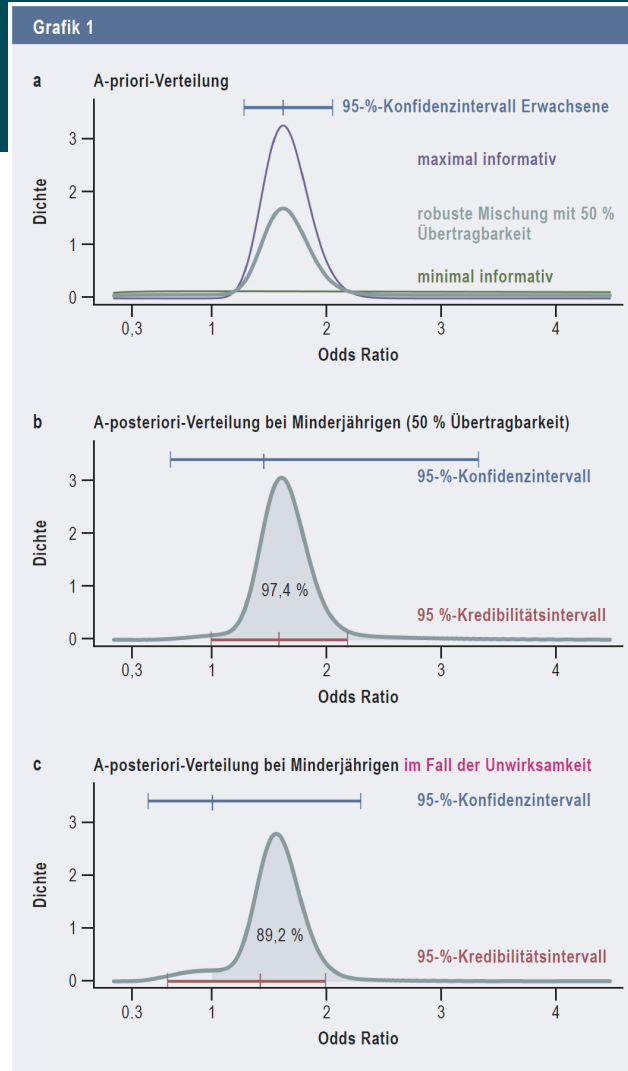
$$m \cdot s_p^2 = \frac{SD^2}{2} \rightarrow m \frac{SD^2}{N_p} = \frac{SD^2}{2} \rightarrow m = \frac{N_p}{2}$$

“
...

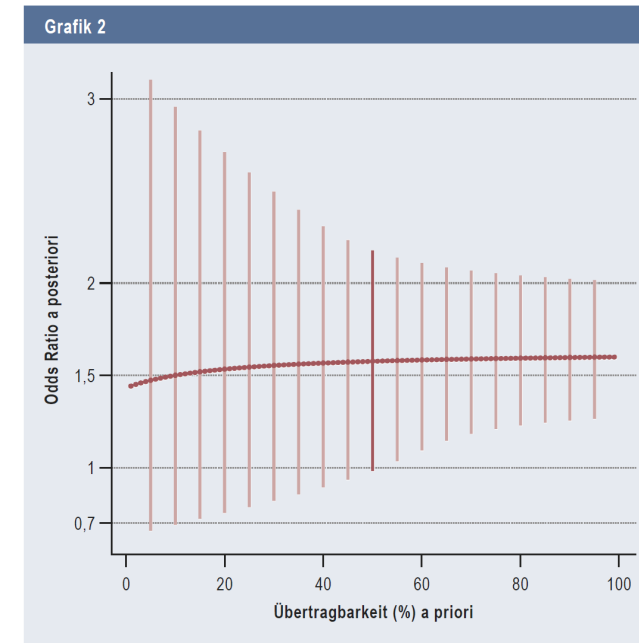
```
library(RBest)
prior <- mixnorm(inf1 = c(w, log(or.a), se.a),
                inf2 = c(1 - w, 0, sqrt(m) * se.k))
posterior <- postmix(prior, m = log(or.k), se = se.k)
prior.density <- dmix(prior, OR.values)
lcl_prior <- exp(qmix(prior, 0.025))
prior_mean <- exp(summary(prior) ["mean"])
```

Third try

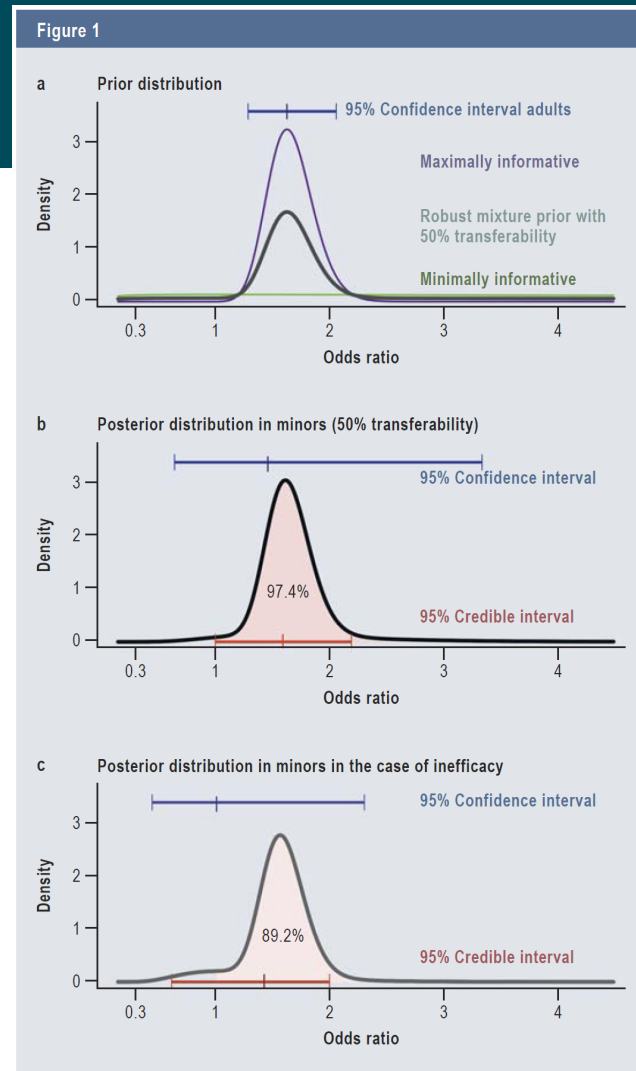




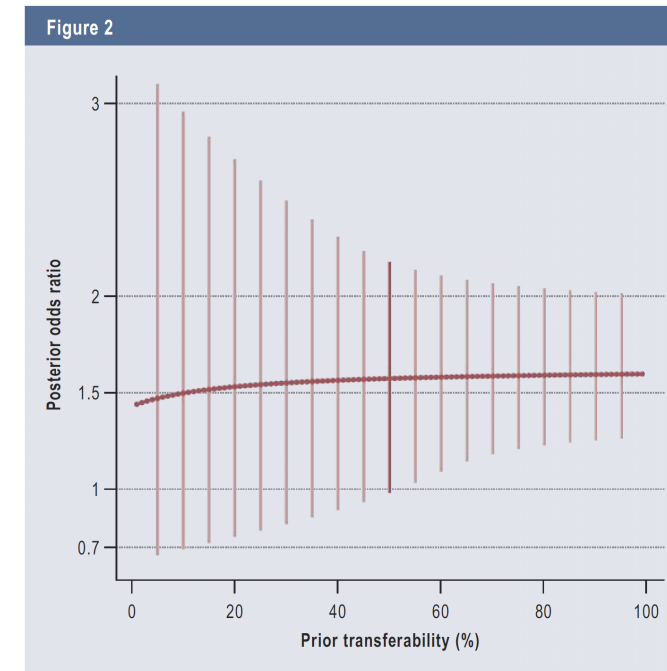
Bayes-Analyse der Odds Ratios (OR) von Belimumab bei Minderjährigen. a: Das Konfidenzintervall für das OR bei Erwachsenen (blau) wird in eine maximal informative A-priori-Verteilung (violett) und eine minimal informative A-priori-Verteilung vom Gewicht einer einzigen Beobachtung an Minderjährigen umgerechnet (grün). Beide Komponenten werden zu einer robusten Mischverteilung kombiniert (dunkelgrau). b: OR bei Minderjährigen. Konfidenzintervall (blau), A-posteriori-Dichte (schwarz), Wahrscheinlichkeit einer Odds Ratio > 1 und Kreditibilitätsintervall (rot). c: theoretisches Szenario, in dem das beobachtete OR bei Minderjährigen genau 1 ist mit Konfidenzintervall (graublau), A-posteriori-Dichte (grau), Wahrscheinlichkeit einer Odds Ratio > 1 und Kreditibilitätsintervall (graurot).



Wenn-Dann-Diagramm. A-posteriori-Kreditibilitätsintervalle der Odds Ratio (rosa Linien) je nach der Übertragbarkeit des Wissens über Erwachsene auf Minderjährige, dem a priori angenommenen Anteil der informativen Komponente in der Mischung. Erwartungswerte der A-posteriori-Verteilungen (dunkelrot). Je höher die Übertragbarkeit des Vorwissens, desto mehr fließt es in die A-posteriori-Verteilung ein und desto genauer kann man die Odds Ratio bei Minderjährigen schätzen.



Bayesian analysis of the odds ratio (OR) for belimumab in minors. a: The confidence interval for the OR in adults (blue) is converted into a maximally informative prior distribution (violet) and a minimally informative prior distribution with weight equivalent to a single observation in minors (green). The two components are combined to yield a robust mixture prior (dark gray). b: The OR in minors. Confidence interval (blue), posterior density (black), probability of an OR > 1, and credible interval (red). c: Theoretical scenario in which the observed OR in minors is exactly 1, with confidence interval (gray-blue), posterior density (gray), probability of an OR > 1, and credible interval (gray-red).



If-then diagram. The posterior credible interval of the odds ratio (pink lines) depending on the transferability of the knowledge regarding adults to minors, the a priori assumed informative component of the mixture. Expected values of the posterior distribution (dark red). The higher the transferability of the prior knowledge, the more it feeds into the posterior distribution and the more precisely the odds ratio in minors can be estimated.

- ▶ What is the origin of Bayesian statistics?
- ▶ How do Bayesian procedures differ from *meta-analyses*?
- ▶ Are prior data also used in *frequentist* statistical testing?
- ▶ Do the results of a Bayesian procedure with a non-informative prior distribution agree with those of a *frequentist* analysis?
- ▶ Can *multiple tests* also cause a problem in Bayesian statistics?
- ▶ What has to be considered when *assessing a published study* in which the Bayesian method was used for analysis?

- ▶ Phase-II-trial
- ▶ Platform trial HEALY ALS
- ▶ Meta-analysis
- ▶ Nowcasting, disease mapping

- ▶ 95%-confidence interval
- ▶ 95%-credible interval
- ▶ Density function
- ▶ Informative prior
- ▶ Meta-analysis
- ▶ Non-informative prior
- ▶ Odds
- ▶ Odds Ratio
- ▶ Posterior distribution
- ▶ Prior distribution
- ▶ Robust mixture prior
- ▶ Unit information prior

1. Title and abstract sent to data base for indexing
2. German version proofed
3. CME: multiple choice questions and answers (them, us)

Questions on the article in issue 10/2025:

Introduction to Bayesian Statistics

The submission deadline is 15 May 2026. Only one answer is possible per question. Please select the answer that is most appropriate.

4. English version
 - a) Abstract
 - b) Struggle with translator re. jargon

Question 1

Which of the following is not possible with Bayesian statistics?

- a) Combination of prior knowledge from earlier studies with new study data
- b) More accurate determination of a p-value
- c) Weighting of prior data for inclusion in new analyses
- d) Combination of data from studies with different evidence levels
- e) Partial integration of data from earlier studies into new analyses

- ▶ From first submission to last edits:
Editor was available, constructive
- ▶ Publisher provides seasoned translators, educators

	1st	2nd	3rd
Simple model	✓	✓	
Intuitive effect measure	✓	✓	
Prior from real experience	✓		✓
Relevance to practitioners			✓
Builds upon diagnostic test		✓	
No formulae	-		✓
Simple diagram	-		✓

- ▶ Relevance beats simple formulae that are not shown
- ▶ Real example beats enabling
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- ▶ If-then diagram
- ▶ Retired physicians do not understand

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